B3 - Games, Graphs and Algebra : Mid-Semester Exam

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September 16, 2024. Time : 10.00 - 12.30 PM. **Maximum points : 15**

2 points will be deducted if you do not write your name on the answerscript.

There are two parts to the question paper - PART A and PART B. Read the instructions for each part carefully.

1 PART A : MULTIPLE-CHOICE QUESTIONS - 5 Points.

Please write only TRUE or FALSE in your answer scripts. No explanations are needed. Write PART A answers in a separate page.

1. $\mathrm{Pic}(G) \cong \mathbb{Z}^2 \times \mathrm{Jac}(G)$.
2. There exists a graph G, divisor D and firing script σ such that $\deg(D - \mathrm{div}(\sigma)) > \deg(D)$.
3. For all graphs G and any $q \in V$, we have that $\deg(v)[v - q] = \sum_{w \in V : w \sim v}[w - q]$.
4. The genus g is at most $\binom{n-1}{2}$ for a connected graph on n vertices.
5. There exists a graph G such that $Jac(G) = \mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_4$.

2 PART B : 10 Points.

ALL QUESTIONS CARRY 5 POINTS. ATTEMPT ANY TWO OF THEM.

You are free to use any results that you have learnt in your course but please cite them clearly. Provide as many details as you can.

1. Let C_n denote the cycle graph with vertices $v_0, \ldots, v_{n-1}$ arranged counter-clockwise in a circle. Identify each divisor D with a vector $(D(v_0), \ldots, D(v_{n-1}))$. Show that the two divisors D and D' on C_n of the same degree are linearly equivalent iff $D \cdot (0, \ldots, n-1) = D' \cdot (0, \ldots, n-1) \bmod n$.
2. Let $D \sim D'$. Show that D is a maximal unwinnable divisor iff D' is a maximal unwinnable divisor. If D is a unwinnable divisor, show that there exists a maximal unwinnable divisor N such that $N \geq D$.
3. Consider the house graph as below. Compute the q-reduced divisors that are linearly equivalent to $D_1 = (-3, 2, 4, -2, 1)$ and $D_2 = (2, 1, -5, 2, 2)$ respectively. Here we have represented divisor D as a vector $(D(q), D(v2), D(v3), D(v4), D(v5))$.